

Unit-I

Number Systems, Binary Codes, Logic gates

Digital systems: A digital system is the one that handles only discrete values or signals.

The word digital describes any system based on discontinuous data or events.

Digital is the method of storing, processing and transmitting information through the use of discrete electronic pulses that represents binary digits 0 & 1.

There is no in between values

Advantages:

1. Noise margin (resistance to noise) robustness
2. Error correction and detection
3. Easily programmable
4. Cheap electronic devices circuits.

Disadvantages:

1. Analog to digital conversion is required.
2. Uses more energy.

Number Systems: A digital system can understand positional number system only where there are a few

Symbols called digits and these symbols represent different values depending on the position they occupy in the number.

→ The value of each digit in a number can be determined using the digit, the position of the digit in the number and the base of the number system.

Base or radix : Base is defined as the total no. of symbols available in the number system.

Types of number systems:

1. Positional number systems

2. non-positional weighted number systems

1. Positional weighted number systems:

Ex:- decimal, binary, octal, hexadecimal

1. decimal number system - base is 10. (0,1,2,3,4,5,6,7,8,9)

2. binary number system - base is 2. (0,1)

3. octal number system - base is 8. (0,1,2,3,4,5,6,7)

4. Hexa decimal number system - base is 16. (0,1,2,3,4,5,6,7,8,9, A, B, C, D, E, F)

Eg for all 4 decimal systems

$$\begin{array}{ccccccc} (A)_{16} & = & (10)_{10} & = & (1010)_2 & = & (12)_8 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{hexa decimal} & & \text{decimal} & & \text{binary} & & \text{octal} \end{array}$$

Decimal number system: Mixed decimal number is represented

$$\text{by, } \underbrace{\alpha_n \alpha_{n-1} \alpha_{n-2} \dots \alpha_3 \alpha_2 \alpha_1 \alpha_0}_{\text{integer}} \cdot \underbrace{\alpha_{-1} \alpha_{-2} \alpha_{-3} \dots \alpha_{-k}}_{\text{fraction}}$$

decimal point

It is a positional weighted number system. It has 10 unique symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

The base or radix is 10

For any decimal number, the representation is

$$(\alpha_{n-1} \times 10^{n-1}) + \dots + (\alpha_2 \times 10^2) + (\alpha_1 \times 10^1) + (\alpha_0 \times 10^0) + (\alpha_{-1} \times 10^{-1}) + (\alpha_{-2} \times 10^{-2}) + \dots + (\alpha_{-k} \times 10^{-k})$$

$$\begin{aligned} \text{Eg:- } 454.76_{10} &= (4 \times 10^2) + (5 \times 10^1) + (4 \times 10^0) + (7 \times 10^{-1}) + (6 \times 10^{-2}) \\ &= 400 + 50 + 4 + 0.7 + 0.06 \\ &= 454.76_{10} \end{aligned}$$

Binary Number System:

It is a positional weighted number system

It has two unique symbols (0, 1)

The base or radix is 2.

Mixed binary number is given by

$$\alpha_{n-1} \alpha_{n-2} \dots \alpha_2 \alpha_1 \alpha_0 \cdot \alpha_{-1} \alpha_{-2} \alpha_{-3} \dots \alpha_{-k}$$

binary point

For any given binary number the representation in decimal form is

$$(\alpha_{n-1} \times 2^{n-1}) + \dots + (\alpha_2 \times 2^2) + (\alpha_1 \times 2^1) + (\alpha_0 \times 2^0) + (\alpha_{-1} \times 2^{-1}) + (\alpha_{-2} \times 2^{-2}) + \dots + (\alpha_{-k} \times 2^{-k})$$

Eg:- $1001.1 = (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1})$

$$= 8 + 1 + 0.5$$

$$= 9.5_{10}$$

Octal number system:

→ It is a positional weighted system

→ It has 8 unique symbols (0, 1, 2, 3, 4, 5, 6, 7)

→ The base or radix is 8.

Mixed octal number is given by

$$\underbrace{\alpha_{n-1} \alpha_{n-2} \dots \alpha_2 \alpha_1 \alpha_0}_{\text{Integer}} \cdot \underbrace{\alpha_{-1} \alpha_{-2} \alpha_{-3} \dots \alpha_{-k}}_{\text{fraction}}$$

Octal point

For any given octal number the representation in decimal form is

$$(\alpha_{n-1} \times 8^{n-1}) + \dots + (\alpha_2 \times 8^2) + (\alpha_1 \times 8^1) + (\alpha_0 \times 8^0) + (\alpha_{-1} \times 8^{-1}) + \dots + (\alpha_{-k} \times 8^{-k})$$

Eg:- $13_8 = 1 \times 8^1 + 3 \times 8^0 = 8 + 3 = 11_{10}$

* In binary number system 3 bits or binary digits is equal to one octal digit

Hexa decimal number system:

→ It is a positional weighted number system

→ It has 16 unique symbols (0, 1, ..., 9, A, B, C, D, E, F)

Hexadecimal number is given by

$$\underbrace{\alpha_{n-1} \alpha_{n-2} \dots \alpha_2 \alpha_1 \alpha_0}_{\text{Integer}} \underbrace{\alpha_{-1} \alpha_{-2} \alpha_{-3} \dots \alpha_{-k}}_{\substack{\text{Hexadecimal} \\ \text{fraction} \\ \text{point}}}$$

→ The base or radix is 16

For any given hexadecimal number, the representation in decimal form is

$$(\alpha_{n-1} \times 16^{n-1}) + \dots + (\alpha_2 \times 16^2) + (\alpha_1 \times 16^1) + (\alpha_0 \times 16^0) + (\alpha_{-1} \times 16^{-1}) + \dots + (\alpha_{-k} \times 16^{-k})$$

$$\text{Eg:- } (100)_{16} = 1 \times 16^2 + 0 \times 16 + 0 \times 16^0 = 256_{10}$$

$$(6A)_{16} = 6 \times 16 + A \times 16^0 = 96 + 10 = 106_{10}$$

Conversion from one number system to another

Binary counting

decimal no	binary no
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000

decimal no	binary no
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000
17	10001
18	10010
19	10011

Binary addition

$$\begin{array}{r} 0 \\ +0 \\ \hline 0 \end{array} \quad \begin{array}{r} 1 \\ +0 \\ \hline 1 \end{array} \quad \begin{array}{r} 0 \\ +1 \\ \hline 1 \end{array} \quad \begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$$

carry sum

MSB $\rightarrow$ 10101 $\rightarrow$ LSB
 most last significant bit
 significant bit

Octal counting

decimal no	octal no
0	00
1	01
2	02
3	03
4	04
5	05
6	06
7	07
8	10
9	11

decimal no	octal no
10	12
11	13
12	14
13	15
14	16
15	17
16	20
17	21
18	22
19	23

Hexa decimal counting

decimal no	hexa decimal no
0	00
1	01
2	02
3	03
4	04
5	05
6	06
7	07
8	08
9	09

decimal no	hexa decimal
10	0A
11	0B
12	0C
13	0D
14	0E
15	0F
16	10
17	11
18	12
19	13
20	14

Conversion of any radix to decimal

1. Binary to decimal:

Eg:- $(1101)_2$
 $2^3 \ 2^2 \ 2^1 \ 2^0$

$$= (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$= 8 + 4 + 1$$

$$= 13_{10}$$

ii) $(11001.010)_2 = (1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3})$
 $2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \ 2^{-1} \ 2^{-2} \ 2^{-3}$

$$= 16 + 8 + 0 + 0 + 1 + 0 + 0.25 + 0$$

$$= 25.25_{10}$$

iii) $(010111.001)_2 = (0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3})$
 $2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \ 2^{-1} \ 2^{-2} \ 2^{-3}$

$$= 32 + 8 + 0 + 2 + 1 + 0.125$$

$$= 43.125_{10}$$

iv) $(0.111)_2 = (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3})$
 $2^0 \ 2^{-1} \ 2^{-2} \ 2^{-3}$

$$= 0 + 0.5 + 0.25 + 0.125$$

$$= 0.875_{10}$$

2. Octal to decimal

Eg:- $(27)_8 = (2 \times 8^1 + 7 \times 8^0)$

$$= 16 + 7$$

$$= 23_{10}$$

ii) $(245)_8 = (2 \times 8^2 + 4 \times 8^1 + 5 \times 8^0)$

$$= 2 \times 64 + 32 + 5$$

$$= 165_{10}$$

iii) $(0.52)_8 = (0 \times 8^0 + 5 \times 8^{-1} + 2 \times 8^{-2}) = 0.65625_{10}$

$$\begin{aligned} \text{iii } (3462)_8 &= (3 \times 8^3 + 4 \times 8^2 + 6 \times 8^1 + 2 \times 8^0) \\ &= 24 + 272 \\ &= 296_{10} \end{aligned}$$

$$\begin{aligned} \text{iv } (266)_8 &= (2 \times 8^2 + 6 \times 8 + 6 \times 8^0) \\ &= 182_{10} \end{aligned}$$

Hexadecimal to decimal

$$\begin{aligned} \text{i } (FF)_{16} &= (F \times 16^1) + (F \times 16^0) \\ &= 15 \times 16 + 15 \\ &= 255_{10} \end{aligned}$$

$$\begin{aligned} \text{ii } (101)_{16} &= 1 \times 16^2 + 0 \times 16^1 + 1 \times 16^0 \\ &= 256 + 1 \\ &= 257_{10} \end{aligned}$$

$$\begin{aligned} \text{iii } (ABC)_{16} &= A \times 16^2 + B \times 16 + C \times 16^0 \\ &= 10 \times 256 + 11 \times 16 + 12 \\ &= 2748_{10} \end{aligned}$$

$$\begin{aligned} \text{iv } (0.3D)_{16} &= 0 \times 16^0 + 3 \times 16^{-1} + 13 \times 16^{-2} \\ &= 0 + 0.238 \\ &= 0.2382_{10} \end{aligned}$$

$$\begin{aligned} \text{v } (AB.C)_{16} &= 10 \times 16 + 11 \times 16^0 + C \times 16^{-1} \\ &= 160 + 11 + 12 \times 16^{-1} \\ &= 171.75_{10} \end{aligned}$$

$$\begin{aligned} \text{vi } (DD.8)_{16} &= 13 \times 16 + 13 \times 16^0 + 8 \times 16^{-1} \\ &= 221.81_{10} \end{aligned}$$

Conversion of decimal to any radix

Double Dabble method

Integer - successive division method

Fraction - successive multiplication method

Eg:- $(26.75)_{10} = (?)_2$

For integer part

$$\begin{array}{r} 26 = 2 \overline{) 26} \\ \underline{2 } 13 - 0 \rightarrow \text{LSB} \\ \underline{2 } 6 - 1 \\ \underline{2 } 3 - 0 \\ \underline{2 } 1 - 1 \\ \text{MSB} \leftarrow \end{array}$$

$$26 = 11010_2$$

For fraction part

$$\begin{aligned} 0.75 &= 0.75 \times 2 = 1.50 \quad \text{M.S.B} \\ 1.50 &= 1.00 + 0.50 \\ 0.50 &\times 2 = 1.00 \\ 1.00 &= 1.00 + 0.00 \end{aligned}$$

$$(26.75)_{10} = (11010.11)_2$$

i) $244_{10} = (?)_2$

$$\begin{array}{r} 244 = 2 \overline{) 244} \\ \underline{2 } 122 - 0 \\ \underline{2 } 61 - 0 \\ \underline{2 } 30 - 1 \\ \underline{2 } 15 - 0 \\ \underline{2 } 7 - 1 \\ \underline{2 } 3 - 1 \\ \underline{2 } 1 - 1 \end{array}$$

$$244 = 11110100_2$$

$$244_{10} = (11110100)_2$$

ii) $26.025_{10} = (?)_2$

$$\begin{aligned} 26 &= 11010_2 \\ 0.025 &= 0.025 \times 2 = 0.50 \\ 0.50 &\times 2 = 1.00 \\ 1.00 &= 1.00 + 0.00 \end{aligned}$$

$$\begin{aligned} 0.025 &\times 2 = 0.05 \\ 0.05 &\times 2 = 0.1 \\ 0.1 &\times 2 = 0.2 \\ 0.2 &\times 2 = 0.4 \\ 0.4 &\times 2 = 0.8 \\ 0.8 &\times 2 = 1.6 \end{aligned}$$

$$0.025 = 000001$$

$$26.025 = (11010.000001)_2$$

$$\text{iii } 37.25_{10}$$

$$\begin{array}{r} 37 = 2 \overline{) 37} \\ \underline{2 \ 18} - 0 \\ \underline{2 \ 9} - 0 \\ \underline{2 \ 4} - 0 \\ \underline{2 \ 2} - 0 \\ 1 - 0 \end{array}$$

$$37 = 10010100101$$

$$0.25 = 0.25 \times 2 = 0.50$$

$$0.50 \times 2 = 1.00$$

$$0.00 \times 2 = 0.00$$

$$0.25 = 0.01010 \Rightarrow 37.25_{10} = 100101.010$$

$$\text{iv } (52)_{10}$$

$$\begin{array}{r} 52 = 2 \overline{) 52} \\ \underline{2 \ 26} - 0 \\ \underline{2 \ 13} - 0 \\ \underline{2 \ 6} - 0 \\ \underline{2 \ 3} - 0 \\ 1 - 1 \end{array}$$

$$52_{10} = (110100)_2$$

Decimal to Octal

$$\text{i) } (160)_{10}$$

$$\begin{array}{r} 160 = 8 \overline{) 160} \\ \underline{8 \ 80} - 0 \\ \underline{8 \ 20} - 0 \\ \underline{8 \ 2} - 0 \end{array}$$

$$160 = (240)_8$$

$$\text{ii } (0.52)_{10} = (?)_8$$

$$0.52 = 0.52 \times 8 = 4.16$$

$$0.16 \times 8 = 1.28$$

$$0.28 \times 8 = 2.24$$

$$0.24 \times 8 = 1.92$$

$$0.92 \times 8 = 7.36$$

$$0.36 \times 8 = 2.88$$

$$(0.52)_{10} = (0.412172)_8$$

$$\text{iii } 756.32_{10} = (?)_8$$

$$\begin{array}{r} 756 = 8 \overline{) 756} \\ \underline{8 \ 94} - 4 \\ \underline{8 \ 11} - 6 \\ 1 - 3 \end{array}$$

$$0.32 = 0.32 \times 8 = 2.56$$

$$0.56 \times 8 = 4.48$$

$$0.48 \times 8 = 3.84$$

$$0.84 \times 8 = 6.72$$

$$0.72 \times 8 = 5.76$$

$$0.32 = 0.24365$$

$$756.32_{10} = (1364.24365)_8$$

$$iv \quad (45.23)_{10} = ()_8$$

$$45 = \begin{array}{r} 8 \overline{) 45} \\ \underline{5} \\ 5 \\ \underline{5} \\ 0 \end{array}$$

$$45 = 55$$

$$0.23 = 0.23 \times 8 = 1.84$$

$$0.84 \times 8 = 6.72$$

$$0.72 \times 8 = 5.76$$

$$0.76 \times 8 = 6.08$$

$$0.23 = 0.1656$$

$$(45.23)_{10} = (55.1656)_8$$

Decimal to hexadecimal:

$$i \quad (255)_{10} = ()_{16}$$

$$16 \overline{) 255} \\ \underline{15} \\ 15 $$

$$16 = 15 + 1 = (FF)_{16}$$

$$ii \quad (45.732)_{10}$$

$$45 = 16 \overline{) 45}$$

$$45 = 2D$$

$$0.732 = 0.732 \times 16 = 11.712$$

$$0.712 \times 16 = 11.392$$

$$0.392 \times 16 = 6.272$$

$$0.272 \times 16 = 4.352$$

$$0.352 \times 16 = 5.632$$

$$0.732 = (BB645)$$

$$(45.732)_{10} = (2D.BB645)_{16}$$

$$v \quad 0.078_{10} = ()_8$$

$$0.078 = 0.078 \times 8 = 0.624$$

$$= 0.624 \times 8 = 4.992$$

$$= 0.992 \times 8 = 7.936$$

$$= 0.936 \times 8 = 7.488$$

$$= 0.488 \times 8 = 3.904$$

$$0.078 = (0.04773)_8$$

$$iii \quad (2598)_{10} = ()_{16}$$

$$2598 = 16 \overline{) 2598} \\ \underline{16} \\ 10 $$

$$2598 = (A26)_{16}$$

$$iv \quad (0.6752)_{10}$$

$$0.6752 = 0.6752 \times 16 = 10.8032$$

$$= 0.8032 \times 16 = 12.8512$$

$$= 0.8512 \times 16 = 13.6192$$

$$= 0.6192 \times 16 = 9.9072$$

$$= 0.9072 \times 16 = 14.5152$$

$$(0.6752)_{10} = (A.CD9E)_{16}$$

Binary to Octal: -

Base of octal number system $8 = 2^3$

group of 3 binary bits = 1 octal digit

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Eg:- $(1110011)_2 = (163)_8$

$= 001\ 110\ 011$
 $= 1\ 6\ 3$

$\rightarrow (11101.0011)_2 = 8P26$

$011\ 101\ 0001\ 0100$

$3\ 5\ 0\ 4$

$\therefore (11101.0011) = 35.0314$

$\rightarrow (10101.111)_2$

$= 010\ 101\ 111\ 100$

$2\ 5\ 7\ 4$

$\therefore (10101.111)_2 = 25.74$

$\rightarrow (10.01)_2$

$= 010\ 010$

$= 2\ 2$

$\therefore (10.01)_2 = (2.2)_8$

$\rightarrow (0010.0010000111)_2$

$= 000\ 010\ 001\ 000\ 011\ 100$

$= 02.1034$

$\therefore (0010.0010000111)_2 = 02.1034$

$\rightarrow (0.0100)_2$

$000\ 010\ 000$

$0\ 2\ 0$

$\therefore (0.0100)_2 = 0.2$

$(0.0100)_2 = 0.2$

$(0.0100)_2 = 0.2$

Binary to hexadecimal

Base of hexadecimal number system $16 = 2^4$

group of 4 binary bits = 1 hexadecimal digit

hexadecimal	binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
(A) 10	1010
B	1011
C	1100
D	1101
E	1110
F	1111

$$\text{Eg: } (1110011)_2$$

$$= 0111 \quad 0011$$

$$= 7 \quad 3$$

$$\therefore (1110011)_2 = (73)_{16}$$

$$\rightarrow (111010011)_2$$

$$= 0001 \quad 1101 \quad 0011$$

$$= 1 \quad D \quad 3$$

$$\therefore (111010011)_2 = (1D3)_{16}$$

$$\rightarrow (101011111)_2$$

$$= 0001 \quad 0101 \quad 1111$$

$$= 1 \quad 5 \quad F$$

$$\therefore (101011111)_2 = (15F)_{16}$$

$$\rightarrow (1001)_2$$

$$= 0010 \quad 0100$$

$$= 2 \cdot 4$$

$$\therefore (1001)_2 = (2.4)_{16}$$

$$\rightarrow (0010.001000011)_2$$

0010	0010	0001	1100
2	2	1	2

$$\therefore (0010.001000011)_2 = (2.21C)_{16}$$

$$\rightarrow (0.0100)_2$$

0000	0100
0	4

$$(0.0100)_2 = (0.4)_{16}$$

Decimal to Binary

$$\text{Eg:- } (25.76)_8$$

$(25.76) =$	2	5	7	6
	↓	↓	↓	↓
	010	101	111	0110

$$(25.76)_8 = (010101.11110)_{10}$$

$$\rightarrow (6574.25)_8$$

$=$	6	5	7	4	2	5
$=$	110	101	111	100	010	101

$$= (11010111100.010101)_{10}$$

$$\rightarrow (0.763)_8$$

$=$	0	7	6	3
$=$	000	111	110	011

$$= (0.11110011)_2$$

$$\rightarrow (12345)_8$$

$=$	1	2	3	4	5
$=$	001	010	011	100	101

$$= (00101001100101)_2$$

Hexadecimal to binary

$$\rightarrow (FAC.35)_{16}$$

$$= \begin{matrix} F & A & C & 3 & 5 \\ = 1111 & 1010 & 1100 & 0011 & 0101 \end{matrix}$$

$$= (111110101100.00110101)_2$$

$$\rightarrow (12AB)_{16}$$

$$= \begin{matrix} 1 & 2 & A & B \\ = 0001 & 0010 & 1010 & 1011 \end{matrix}$$

$$= (0001001010101011)_2$$

$$\rightarrow (0.012)_{16}$$

$$= \begin{matrix} 0 & 0 & 1 & 2 \\ = 0 & 0000 & 0001 & 0010 \end{matrix}$$

$$= (0.00000001001)_2$$

$$\rightarrow (AB.58)_{16}$$

$$= \begin{matrix} A & B & 5 & 8 \\ = 1010 & 1011 & 0101 & 1000 \end{matrix}$$

$$= (10101011.0101)_2$$

Octal to hexadecimal

$$0 \leftrightarrow H$$

$$\rightarrow (25.434)_8$$

$$\begin{matrix} 2 & 5 & 4 & 3 & 4 \\ 010 & 101 & 100 & 011 & 100 \end{matrix}$$

$$= (10101 \cdot 100011)_2$$

$$2^{(101001100100)} =$$

$$= 0001 \ 0101 \cdot \ 1000 \ 1110$$

$$= 1 \ 5 \cdot \ 8 \ E$$

$$= (15 \cdot 8E)_{16}$$

$$\rightarrow (57 \cdot 34)_8$$

$$= 5 \ 7 \ 3 \ 4$$

$$= 101 \ 111 \ 011 \ 100$$

$$= (101111 \cdot 011100)_2$$

$$= 0010 \ 1111 \ 0111 \ 0000$$

$$= 2 \ F \cdot \ 37$$

$$= (2F \cdot 37)_{16}$$

$$\rightarrow (0 \cdot 774)_8$$

$$= 0000 \ 7 \ 7 \ 4$$

$$= 0 \ 0111 \ 0111 \ 100$$

$$= (0 \cdot 11111100)_2$$

$$= 0 \ 1111 \ 1110$$

$$= 0 \ FE$$

$$= (0 \cdot FE)_{16}$$

$$\rightarrow (1754)_8$$

$$= 1 \ 7 \ 5 \ 4$$

$$= 0001 \ 111 \ 101 \ 100$$

$$= (1111101100)_2$$

$$= 1111 \ 1011 \ 0011 \ 1110 \ 1100$$

$$= FB \ 3 \ E \ C$$

$$= (FB)_{16} \ (3EC)_{16}$$

$$2^{(25 \cdot 0A7)} \leftarrow$$

$$\begin{matrix} 2 & 5 & 0 & A & 7 \\ 1010 & 1100 & 0011 & 0101 & 1111 \end{matrix} =$$

$$2^{(10101100 \cdot 00110101111)} =$$

$$2^{(8AB1)} \leftarrow$$

$$\begin{matrix} 8 & A & B & 1 \\ 1101 & 0101 & 0100 & 1000 \end{matrix} =$$

$$2^{(1101010101001000)} =$$

$$2^{(010 \cdot 0)} \leftarrow$$

$$\begin{matrix} 0 & 1 & 0 & 0 \\ 0100 & 1000 & 0000 & 0 \end{matrix} =$$

$$2^{(100100000000 \cdot 0)} =$$

$$2^{(22 \cdot 9A)} \leftarrow$$

$$\begin{matrix} 2 & 2 & 9 & A \\ 0001 & 1010 & 1101 & 0101 \end{matrix} =$$

$$2^{(11010 \cdot 11010101)} =$$

$$\begin{matrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1000 & 1010 & 1101 & 0101 & 1000 \end{matrix}$$

$$H \leftarrow 8 \rightarrow 0$$

$$2^{(454 \cdot 3E)} \leftarrow$$

$$\begin{matrix} 4 & 5 & 4 & 3 & E \\ 001 & 110 & 001 & 101 & 110 \end{matrix}$$

Hexadecimal to decimal

$H \xleftrightarrow{8} O$

$(5AC)_{16}$

5	A	C
01001	1010	1100
01000	110	101 100
2	6	5 4

$= (2654)_8$

$\rightarrow (A125)_{16}$

A	1	2	5
1010	0001	0010	0101
101	000	010	010 010 100
5	0 2224		

$= (50224)_8$

$\rightarrow (2018)_{16}$

2	0	1	8
0010	0000	0001	1000
001	000	000	000 000 000
1	00140		

$= (100140)_8$

$$= (10101 \cdot 1000111)_2$$

$$= 0001 \ 0101 \ 1000 \ 1110$$

$$= 1 \ 5 \ 8 \ E$$

$$= (158E)_{16}$$

$$\rightarrow (5734)_8$$

$$= 5 \ 7 \ 3 \ 4$$

$$= 101 \ 111 \ 011 \ 100$$

$$= (10111 \cdot 011100)_2$$

$$= 0010 \ 1111 \ 0111 \ 0000$$

$$= 2 \ F \ 7$$

$$= (2F7)_{16}$$

$$\rightarrow (0774)_8$$

$$= 0 \ 7 \ 7 \ 4$$

$$= 0 \ 111 \ 111 \ 100$$

$$= (011111100)_2$$

$$= 0 \ 1111 \ 1110$$

$$= 0 \ E \ F$$

$$= (0EF)_{16}$$

$$\rightarrow (1754)_8$$

$$= 1 \ 7 \ 5 \ 4$$

$$= 0001 \ 111 \ 101 \ 100$$

$$= (00111101100)_2$$

$$= 0011 \ 1110 \ 1100$$

$$= (3EC)_{16}$$

Hexadecimal to decimal

$$H \xleftrightarrow{8} O$$

$(5AC)_{16}$

$$\begin{aligned} & \begin{array}{ccc} 5 & A & C \end{array} \\ & = 0101 \quad 1010 \quad 1100 \\ & = 010 \quad 110 \quad 101 \quad 100 \\ & = 2 \quad 6 \quad 5 \quad 4 \\ & = (2654)_8 \end{aligned}$$

$\rightarrow (A12.50)_{16}$

$$\begin{aligned} & \begin{array}{cccc} A & 1 & 2 & 5 \end{array} \\ & = 1010 \quad 0001 \quad 0010 \quad 0101 \\ & = 101 \quad 000 \quad 010 \quad 010 \quad 010 \quad 100 \\ & = 5 \quad 0 \quad 2 \quad 2 \quad 2 \quad 4 \\ & = (502224)_8 \end{aligned}$$

$\rightarrow (2018)_{16}$

$$\begin{aligned} & = 2 \quad 0 \quad 1 \quad 8 \\ & = 0010 \quad 0000 \quad 0001 \quad 1000 \\ & = 001 \quad 000 \quad 000 \quad 001 \quad 100 \\ & = 1 \quad 0 \quad 0 \quad 1 \quad 4 \\ & = (10014)_8 \end{aligned}$$

Conversion of any radix to radix

$$(12)_3 \xrightarrow{0} ()_4$$

$$12 = 1 \times 3^1 + 2 \times 3^0$$

$$= 3 + 2$$

$$= 5_{10}$$

$$5_{10} = 4 \frac{5}{1-1}$$

$$= (11)_4$$

Binary arithmetic

Binary addition

$$\begin{array}{r} 1100 \rightarrow 12 \\ + 111 \rightarrow 7 \\ \hline 10011 \rightarrow 19 \end{array}$$

Binary subtraction

$$\begin{array}{r} 1101 \rightarrow 13 \\ - 101 \rightarrow 5 \\ \hline 1000 \rightarrow 8 \end{array}$$

$$\begin{array}{r} 1001 \rightarrow 9 \\ - 110 \rightarrow 6 \\ \hline 011 \rightarrow 3 \end{array}$$

$$(58.56)_{10}$$

$$58 = 8 \overline{) 58} \\ \underline{7-2}$$

$$0.56 = 0.56 \times 8 = 4.48$$

$$= 0.48 \times 8 = 3.84$$

$$= 0.84 \times 8 = 6.72$$

$$= 0.72 \times 8 = 5.76$$

$$= 0.76 \times 8 = 6.08$$

$$(58.56)_{10} = (72.43656)_8$$

$$58 = 2 \overline{) 58} \\ \underline{29-0} \\ 2 \overline{) 14} - 1 \\ \underline{7-0} \\ 2 \overline{) 3} - 1 \\ \underline{1-1}$$

$$0.56 = 0.56 \times 2 = 1.12$$

$$0.12 \times 2 = 0.24$$

$$0.24 \times 2 = 0.48$$

$$0.48 \times 2 = 0.96$$

$$(58.56)_{10} = (111010.96)_2$$

$$58 = 16 \overline{) 58} \\ \underline{3-10}$$

$$0.56 = 0.56 \times 16 = 8.96$$

$$0.96 \times 16 = 15.36$$

$$0.36 \times 16 = 5.76$$

$$0.76 \times 16 = 12.16$$

$$0.16 \times 16 = 0.56$$

$$(58.56)_{10} = (3A.8B)_{16}$$

ii (46.58)₁₆

$$46.58 = 4 \times 16 + 6 \times 16^0 + 5 \times 16^{-1} + 8 \times 16^{-2}$$

$$= 64 + 6 + 0.3125 + 0.03125$$

$$= 70.34375_{10}$$

$$\begin{array}{r} 70 = 2 \overline{) 70} \\ \underline{2 \ 35} \ 0 \\ \underline{2 \ 14} \ 1 \\ \underline{2 \ 7} \ 0 \\ \underline{2 \ 3} \ 1 \\ 1 \end{array}$$

$$0.34375 = 0.34375 \times 2 = 0.6875$$

$$0.6875 \times 2 = 1.375$$

$$0.375 \times 2 = 0.75$$

$$0.75 \times 2 = 1.5$$

$$0.5 \times 2 = 1$$

$$0 \times 2 = 0$$

$$= (111010.01011)_2$$

$$\begin{array}{r} 70 = 8 \overline{) 70} \\ \underline{8 \ 56} \ 14 \\ \underline{1 \ 0} \end{array}$$

$$0.34375 = 0.34375 \times 8 = 2.75$$

$$0.75 \times 8 = 6$$

$$0 \times 8 = 0$$

$$= (106.26)_8$$

iii (110111101.00110)₂

$$= 1 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1}$$

$$+ 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + 0 \times 2^{-5}$$

$$= 256 + 128 + 0 + 32 + 16 + 8 + 4 + 1 + 1/8 + 1/16$$

$$= (445.21875)_{10}$$

$$\begin{array}{r} 445 = 8 \overline{) 445} \\ \underline{8 \ 352} \ 93 \\ \underline{6 \ 72} \ 16 \end{array}$$

$$= (675.16)_8$$

$$0.21875 = 0.21875 \times 8 = 1.75$$

$$= 0.75 \times 8 = 6.0$$

$$= 0 \times 8 = 0$$

$$\begin{array}{r} 445 = 16 \overline{) 445} \\ \underline{16 \ 272} \ 173 \\ \underline{1 \ 12} \end{array}$$

$$0.21875 = 0.21875 \times 16 = 3.5$$

$$0.5 \times 16 = 8$$

$$0 \times 16 = 0$$

$$iv \quad (257.12)_8$$

$$= 2 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 + 1 \times 8^{-1} + 2 \times 8^{-2}$$

$$= (175.15625)_{10}$$

$$\begin{array}{r} 175 = 2 \overline{) 175} \\ \underline{2 87} -1 \\ 43 -1 \\ 21 -1 \\ 10 -1 \\ 5 -0 \\ 2 -1 \\ 1 -0 \end{array}$$

$$0.15625 = 0.15625 \times 2 = 0.3125$$

$$0.3125 \times 2 = 0.625$$

$$0.625 \times 2 = 1.25$$

$$0.25 \times 2 = 0.5$$

$$0.5 \times 2 = 1$$

$$175 = (10101111.00101)_2$$

$$\begin{array}{r} 175 = 16 \overline{) 175} \\ \underline{10-15} \end{array}$$

$$0.15625 = 0.15625 \times 16 = 2.5$$

$$= (AF.28)_{16}$$

$$0.5 \times 16 = 8$$

$$0 \times 16 = 0$$

$$v \quad (25)_6$$

$$= 2 \times 6^1 + 5 \times 6^0$$

$$= 12 + 5$$

$$= (17)_{10}$$

$$\begin{array}{r} 17 = 7 \overline{) 17} \\ \underline{2-3} \end{array}$$

$$= (23)_7$$

$$\begin{array}{r} 17 = 4 \overline{) 17} \\ \underline{4-1} \end{array}$$

$$= (41)_4$$

Octal arithmetic

Octal addition

$$\begin{array}{r} 46_8 \rightarrow 100 \ 110 \\ 64_8 \rightarrow 110 \ 100 \\ \hline 132_8 \quad 1011 \ 0102 \\ \quad 13 \quad 2_8 \end{array}$$

Octal subtraction

$$\begin{array}{r} 56_{4+8} \rightarrow 101 \ 110 \\ 75 \quad 111 \ 101 \\ \hline 467_8 \quad 100 \ 110 \ 1112 \\ \quad 4 \quad 6 \quad 7_8 \end{array}$$

Hexa decimal arithmetic

Hexa decimal addition

$$\begin{array}{r} 98 \rightarrow 1001 \ 1000 \\ + 18 \rightarrow 0001 \ 1000 \\ \hline B0_{16} \quad 1011 \ 0000 \ 2 \\ \quad B \quad 0_{16} \end{array}$$

$\begin{array}{r} +1 \\ 10 \\ +1 \\ \hline 11 \rightarrow \text{sum} \\ \downarrow \\ \text{carry} \end{array}$

Hexa decimal subtraction

$$\begin{array}{r} 540_{16} \rightarrow 0101 \ 0100 \ 1112 \\ - 009 \quad 0000 \ 0000 \ 1001 \\ \hline 537_{16} \quad 0101 \ 0011 \ 01112 \\ \quad 5 \quad 3 \quad 7_{16} \end{array}$$

Signed and unsigned numbers representation

Unsigned numbers $\rightarrow$ all positive numbers

Signex $\begin{cases} +ve \text{ - signed magnitude} \\ -ve \end{cases}$

$\rightarrow$ signex magnitude

$(r-1)_s \rightarrow 1's \text{ complement - diminished radix complement}$

$r_s \rightarrow 2's \text{ complement - radix complement}$

Binary Complements:

radix = 2

radix complement $\rightarrow r's \rightarrow 2's$

diminished radix complement $\rightarrow (r-1)'s \rightarrow 1's$

$\rightarrow$ to find diminished radix complement,

$$(r^n - r^m - N)$$

$\rightarrow$ to find radix complement, $(r^n - r^m - N) + 1$

$n \rightarrow$ no. of integer digits

$m \rightarrow$ no. of fraction digits

$r \rightarrow$ radix of the number system

$N \rightarrow$ given number

Eg:- $(1011)_2$ find 1's and 2's complements.

1's complement $\rightarrow (r^n - r^m - N)$

$$n=4 \rightarrow (2^4 - 2^0 - 1011)$$

$$m=0 \rightarrow (16 - 1 - 1011_2)$$

$$r=2 \rightarrow (15)_{10} - (1011)_2$$

$$N=1011 \rightarrow 1111 - 1011$$

$$\rightarrow 0100$$

$$\begin{aligned} 2's \text{ complement} &= 0100 + 1 \\ &= 0101_2 \end{aligned}$$

$$(111011)_2$$

$$n=4 \Rightarrow (r^n - r^m - N)$$

$$m=0 \Rightarrow (2^4 - 2^0 - 1110)$$

$$r=2 \Rightarrow (15)_{10} - (1110)_2$$

$$N=1110 \Rightarrow 1111 - 1110$$

$$\Rightarrow 0001$$

$$1's \text{ complement} = 0001$$

$$2's \text{ complement} = 0001 + 1$$

$$= 0010$$

Decimal complements:

radix complement $\rightarrow$ 10's complement

diminished radix complement $\rightarrow$ 9's complement

Find 9's & 10's complement of given numbers

$$\rightarrow 45_{10}$$

$$\rightarrow 74.01_{10}$$

$$n=2 \quad m=0 \quad r=10 \quad N=45$$

$$n=2 \quad m=2 \quad r=10 \quad N=74.01$$

$$\begin{aligned} 9's \text{ complement} &= (10^2 - 10^0 - 45) \\ &= (100 - 1 - 45) \\ &= (99 - 45) \\ &= 54_{10} \end{aligned}$$

$$\begin{aligned} 9's \text{ complement} &= (10^2 - 10^2 - 74.01) \\ &= (100 - 0.01 - 74.01) \\ &= 25.98 \end{aligned}$$

$$\begin{aligned} 10's \text{ complement} &= 54 + 1 \\ &= 55_{10} \end{aligned}$$

$$\begin{aligned} 10's \text{ complement} &= 25.98 + 1 \\ &= 26.98_{10} \end{aligned}$$

Unsigned arithmetic:

Subtraction by using $(r-1)$'s complement form:

Case i:- Subtraction of smallest number from largest number,

Procedure:

1. Determine $(r-1)$'s complement of subtrahend (smallest number)
2. Add the complement to the minuend (largest number)
3. After the addition the carry will be generated, i.e., known as end around carry (EAC)
4. Remove the carry and add it to the LSB (LSD) of the result.

Eg:- Subtract $85_{10} - 42_{10}$ using 9's complement form

1. 9's complement of 42

$$\begin{array}{r} 99 \\ - 42 \\ \hline 57_{10} \end{array}$$

2. $85 + 57 = 142_{10}$

3. carry is 1

4. Add carry to LSD i.e., 43

Eg:- $1110_2 - 0110_2$ using 1's complement form

1's complement 0110 is 1001

$$\begin{array}{r} 1110 \\ 1001 \\ \hline 10111 \end{array}$$

$$\begin{array}{r} 1110 \\ 1001 \\ \hline 1000 \end{array}$$

$$\text{eg:- } (70)_8 - (52)_8$$

1. 7's complement of 52

$$= (8^2 - 8^0 - 52)$$

$$= (64 - 1 - 52)$$

$$= (77 - 52) = 25$$

$$2. 70 + 25 = 115$$

$$= 15$$

$$+ 1$$

$$= -16$$

$$(1F)_{16} - (13)_{16}$$

15's complement of 13

$$= (8^2 - 8^0 - 13)$$

$$= (64 - 1 - 13)$$

$$= (77 - 13) = 64$$

$$\begin{array}{r} 1F \\ - 13 \\ \hline 64 \\ + 1 \\ \hline 65 \end{array}$$

Case ii:- Subtracting largest number from smallest number

Procedure:

1. Determine 10's complement of subtrahend (largest number)

2. Add the complement to the minuend (smallest number)

3. After the addition no carry will be generated i.e., the result is -ve and it is in the complement form.

4. Take the complement of the obtain result to get the true value and place -ve sign

Eg:- subtract using 10's complement form

$$101_2 - 110_2$$

minuend subtrahend

10's complement of 110 = 001

$$101$$

$$001$$

110 → result in complemented form

10's complement of 110 = -001

$$(7450)_{10} - (8461)_{10}$$

1. 9's complement of 8461

$$9999$$

$$8461$$

$$1538_{10}$$

$$\begin{array}{r} 2 \ 7450 \\ 1538 \\ \hline 8988 \end{array}$$

3. 9's complement of 8988

$$\begin{array}{r} 9999 \\ 8988 \\ \hline 1011 \end{array}$$

Result = -1011

$$(1011)_8 - (52)_8$$

The 7's complement of 52 is

$$= (8^2 - 8^0 - 52)$$

$$= (64 - 1 - 52)$$

$$= (77 - 52)$$

$$= 25$$

$$2. \ 148$$

$$\begin{array}{r} 258 \\ 41 \end{array}$$

3. 7's complement of 41 is

$$= (8^2 - 8^0 - 41)$$

$$= (64 - 1 - 41)$$

$$= 77 - 41$$

$$= -36$$

$$iv \ (20)_{16} - (95)_{16}$$

The 15's complement of 95 is

$$= (16^2 - 16^0 - 95)$$

$$= (256 - 1 - 95)$$

$$= (255 - 95)$$

$$= (FF) - 95$$

$$= 6A$$

$$\begin{array}{r} 16255 \\ 15-15 \end{array}$$

$$\begin{array}{r} 15 \ 15 \\ FF \\ 95 \\ \hline 6A \end{array}$$

$$\begin{array}{r} 20 \\ + 6A \\ \hline 8A \end{array}$$

The 16's complement of $2A_{16}$ is

$$= (16^1 - 16^0 - 2A)$$

$$= (255 - 2A)$$

$$= FF - 2A$$

$$= 75$$

$$\begin{array}{r} 1515 \\ FF \\ - 2A10 \\ \hline 75 \end{array}$$

Find 8's complement of 35_8

8's complement of 35 is

$$= (8^2 - 8^0 - 35) + 1$$

$$= (64 - 1 - 35) + 1$$

$$= (28 - 35) + 1$$

$$= 77 - 35 + 1$$

$$= 43_8$$

Find 16's complement of $1A2_{16}$

16's complement of $1A2_{16}$ is

$$= (16^3 - 16^0 - 1A2) + 1$$

$$= (255 - 1A2) + 1$$

$$= (FF - 1A2) + 1$$

$$= 15D + 1$$

$$= 15E$$

Subtraction using radix complement form

Case:- Subtracting smallest number from largest number

1. Determine r's complement to subtrahend (smallest number)

2. Add the complement to the minuend (largest number)

3. After the addition the end around carry will be

generated. Discard the carry and sum itself is the result

Subtract using 2's complement form

Eg:- $1110_2 - 0111_2$

2's complement of 0111 is 1001

$$\begin{array}{r} 1110 \\ 1001 \\ \hline \text{discard } 1 \text{ } 0111 \\ \text{result} \end{array} = 7$$

$$\begin{array}{r} 0111 \\ + 1 \\ \hline 1000 \end{array}$$

1. $58_{10} - 42_{10}$

10's complement of 42 = $58 + (100 - 100 - 42) =$

$$\begin{array}{r} 58 \\ 58 \\ \hline \text{discard } 116 \\ \text{result} \end{array}$$

2. $14_8 - 10_8$

8's complement of 10 = $(8^2 - 8^0 - 10) + 1$

$$= (64 - 1 - 10) + 1$$

$$= (53 - 10) + 1$$

$$= (43) + 1$$

$$= 44$$

$$\begin{array}{r} 14 \\ 68 \\ \hline 102 \\ \text{result} \end{array}$$

3. $FF_{16} - 70_{16}$

16's complement of 70 = $(16^2 - 16^0 - 70) + 1$

$$(256 - 70) + 1$$

$$= 186 + 1$$

$$= 187$$

$$FF + 90 = 18F$$

result

$$\begin{array}{r} 15 \ 15 \\ FF \\ 70 \\ \hline 8F \end{array}$$

$$8F$$

$$+ 1$$

$$90$$

$$FF$$

$$90$$

$$18F$$

Case ii: Subtracting largest no. from smallest no.

1. Determine r's complement to the subtrahend (largest no.)

2. Add the complement to the minuee (smallest no.)

3. After the addition no carry will be generated

4. Therefore result is -ve take the complement of result to get the true value and place -ve sign

Subtract using 2's complement form $0111 - 1110$

2's complement of $1110 = 0010_2$

$$\begin{array}{r} 0111 \\ + 0010 \\ \hline 1001 \end{array}$$

$$\begin{array}{r} 8 \overline{) 511} \\ 640 \\ \hline 701 \end{array}$$

2's complement of $1001 = -0111_2$

$$8 \ 42_{10} - 60_{10}$$

$$10_8 - 12_8$$

10's complement of $60 = 40$

$$158_{16} - 542_{16}$$

$$\begin{array}{r} 42 \\ + 40 \\ \hline 82 \end{array}$$

10's complement of $82 = 18$

$$158 - 128$$

$$\begin{aligned} 8's \text{ complement of } 12 &= (8^2 - 8^0 - 12) + 1 \\ &= (64 - 1 - 12) + 1 \\ &= (51 - 12) + 1 \\ &= 39 + 1 \\ &= 40 \end{aligned}$$

$$\begin{array}{r} 15 \\ + 66 \\ \hline 81 \end{array}$$

8's complement of $103 = (8^3 - 8^0 - 103) + 1$

$$= (512 - 1 - 103) + 1$$

$$= 409 + 1 = 410$$

Binary Signed arithmetic

Represent using 8 bits

$$+12 \rightarrow 00001100 \rightarrow \text{signed magnitude form}$$

$$-12 \rightarrow 10001100 \rightarrow \text{signed magnitude form}$$

$$-12 \rightarrow 11110011 \rightarrow \text{1's complement form}$$

$$-12 \rightarrow 11110100 \rightarrow \text{2's complement form}$$

Signed binary arithmetic using 2's complement form

1. Determine 2's complement of -ve numbers

2. Add the numbers

3. After the arithmetic operation discard the carry if any

4. If MSB is 1 then the result is -ve and take 2's complement of the magnitude of the result to get the true value

5. If MSB is 0 the result is +ve and the sum itself is the result.

Eg:- $+7 \rightarrow 00001111$

$$+3 \rightarrow 00000101$$

$$+10 \rightarrow 00001010$$

ii) $-7 \rightarrow 10000111 \rightarrow 01111000$

$$+3 \rightarrow 00000101$$

2's complement of -7 = 01111001

$$01111001 + 00000101 = 10000000$$

2's complement of 1111100 is 0000100

$$\begin{array}{r} -A \\ -B \\ \hline \hline \end{array} \approx \begin{array}{r} -A \\ +B \\ \hline \hline \end{array} - \begin{array}{r} -A \\ +B \\ \hline \hline \end{array} \approx \begin{array}{r} -A \\ +(-B) \\ \hline \hline \end{array} \approx \begin{array}{r} +A \\ -B \\ \hline \hline \end{array} \approx \begin{array}{r} +A \\ +B \\ \hline \hline \end{array}$$

$$\begin{array}{r} +A \\ +B \\ \hline \hline \end{array} \approx \begin{array}{r} +A \\ +(-B) \\ \hline \hline \end{array}$$

i) $(-17)_{10} - (-10)_{10}$

$$\begin{array}{r} -17 \rightarrow 10010001 \\ + +10 \rightarrow 00001010 \\ \hline \hline \end{array}$$

$$\begin{array}{r} 1001110 \\ +1 \\ \hline 1011111 \end{array}$$

2's complement of -17 = 11101111

$$\begin{array}{r} 11101111 \\ 00001010 \\ \hline 11110001 \end{array}$$

$$\begin{array}{r} 10000000 \\ (+256) - (+32) \\ + (-32) \end{array}$$

2's complement of 11111001 = 00000111

ii) $(10001100)_2 - (10001111)_2$

$$= (+10001100)_2 + (-10001111)_2$$

2's complement of 10001111 = 11110001

$$\begin{array}{r} 10001100 \\ 11110001 \\ \hline 10111101 \end{array}$$

2's complement of 0111101 = 10000001

$$10001100$$

$$+ 00001111$$

2's complement of 10001100 is = 11110100

$$\begin{array}{r} 11110100 \\ 00001111 \\ \hline 10000011 \end{array}$$

The result is +00000011

$$(+12)_8 - (+11)_8$$

$$(+12)_8 + (-11)_8 = 00001010 + (10001001)$$

2's complement of 10001001 = 01110111

$$\begin{array}{r} 00001010 \\ 01110111 \\ \hline 10000001 \end{array}$$

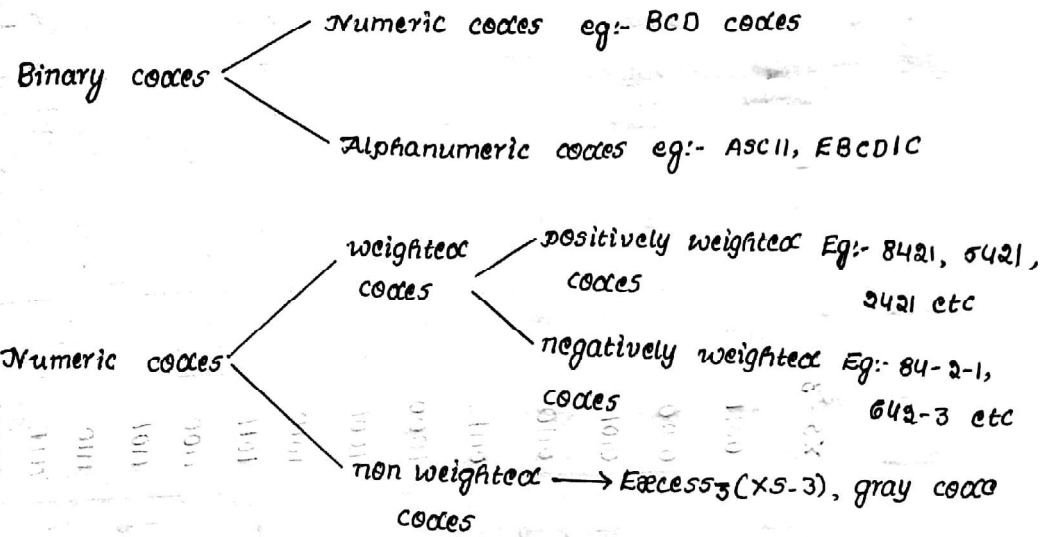
Result is = +00000001

$(25)_{10} - (-10)_{10}$ using 1's complement, 2's complement

$$(25)_{10} - (-10)_{10} = (25)_{10} + (10)_{10}$$

$$\begin{array}{r} (25)_{10} = 00011001 \\ + 10 = 00001010 \\ \hline 00100011 \end{array}$$

Binary Codes:



Decimal digit	8421	2421	5421	642-3	84-2-1	X5-3
0	0000	0000	0000	0000	0000	0000
1	0001	0001	0001	0101	0110	0100
2	0010	0010	0010	0100	0110	0101
3	0011	0011	0100	1001	0101	0110
4	0010	0010	0100	0100	0010	0111
5	1010	1101	0001	1101	1101	0001
6	0110	0011	1001	0110	0101	1001
7	0111	1011	0101	1011	1001	0101
8	0001	0111	1101	0101	0001	1011
9	1001	1111	0011	1111	1111	0011
Unused	0101	1010	1010	1000	1000	1011
612	1101	0110	0110	0100	0100	0111
Combinations	0011	1110	1101	1110	1100	1111
	1101	1001	1101	1000	1100	1000
	0111	1001	1111	1100	1101	1001
	1111	1011	1111	1110	1110	1001

Binary code division arithmetic

→ BCD addition

1. Represent the BCD numbers in BCD form
2. Add the two BCD numbers using ordinary binary addition
3. If four bits sum is equal or less than '9', no correction is required and the sum is in BCD form.
4. If four bits sum is greater than 9 (or) if a carry is generated from four bits sum that sum is invalid
5. To correct this invalid sum add 0110 (6) to the four bit sum

Eg:- 12+15

$$\begin{array}{r}
 12 \rightarrow 0001 \ 0010 \\
 +15 \rightarrow 0001 \ 0101 \\
 \hline
 27 \quad 0010 \ 0111 \\
 \end{array}$$

<9 <9

2 7

48+58

$$\begin{array}{r}
 48 \rightarrow 0100 \ 1110 \quad 0100 \ 1000 \\
 58 \rightarrow 0101 \ 1110 \quad 0101 \ 1000 \\
 \hline
 106 \quad 0000 \quad 10000
 \end{array}$$

carry propagated to next grp

$$\begin{array}{r}
 1010 \ 0006 \\
 0110 \ 0110 \\
 \hline
 0001 \ 0000 \ 0110
 \end{array}$$

(2 groups sums are > 9)

Add correction factor

(0110 to two groups to

skip invalid stages)

(Result in BCD form)

$$24 + 18$$

$$\begin{array}{r} 24 \rightarrow 0010 \quad 00100 \\ + 18 \rightarrow 0001 \quad 1000 \\ \hline 42 \quad 0001 \quad 1100 > 9 \\ \quad 0000 \quad 0110 \\ \quad 0000 \quad 0010 \\ \quad 1 \end{array}$$

4 2

$$175 + 326$$

$$\begin{array}{r} 175 \rightarrow 0001 \quad 0111 \quad 0101 \\ 326 \rightarrow 0011 \quad 0010 \quad 0110 \\ \hline 501 \quad 0100(0) \quad 1001(4) \quad 1011 > 9 \\ \quad 0000 \quad 0000 \quad 0110 \\ \quad 0100(4) \quad 1010 > 9 \quad 0001(1) \\ \quad 0000 \quad 0110 \quad 0000 \\ \quad 0101 \quad 0000 \quad 0001 \\ \quad 5 \quad 0 \quad 1 \end{array}$$

$$589 + 199$$

$$\begin{array}{r} 589 \rightarrow 0101 \quad 1000 \quad 1001 \\ 199 \rightarrow 0001 \quad 1001 \quad 1001 \\ \hline 788 \quad 0111 \quad 1001(0) \quad 1001(11) \\ \quad 0000 \quad 0110 \quad 0110 \\ \quad 0111 \quad 1000 \quad 1000 \\ \quad 7 \quad 8 \quad 8 \end{array}$$

BCD subtraction using complement method

$$20_{10} - 14_{10}$$

$$\begin{array}{r} 20 \xrightarrow{\text{BCD}} 0010 \quad 0000 \\ -14 \xrightarrow{\text{10's complement}} 1000 \quad 0101 \\ \hline \quad 1010 \quad 0101 \\ \quad 0110 \quad 0000 \\ \quad \text{EAC} \quad 10000 \quad 0101 \\ \quad \quad \quad 1 \\ \quad \quad \quad 0000 \quad 0110 \rightarrow 6 \end{array}$$

8-2 (using 10's complement)

$$\begin{array}{r} 8 \xrightarrow{\text{BCD}} 1000 \\ -2 \xrightarrow{\text{10's complement}} 1000 \\ \hline 10000 \end{array}$$

$$\begin{array}{r} 0000 \\ 0110 \\ \hline 0110 \rightarrow 6 \end{array}$$

28-13 using 10's complement

$$\begin{array}{r} 28 \xrightarrow{\text{BCD}} 0010 \quad 1000 \\ -13 \xrightarrow{\text{10's com}} 00 \\ \hline 1000 \quad 0111 \\ \hline 1010 \quad 1111 \\ \hline 0110 \quad 0110 \\ \hline 10001 \quad 0101 \\ \hline 15 \end{array}$$

79-26 using 9's complement

$$\begin{array}{r} 79 \xrightarrow{\text{BCD}} 0111 \quad 1001 \\ -26 \xrightarrow{\text{9's com}} 0010 \quad 0110 \\ \hline 1001 \quad 1111 \\ \hline 0111 \quad 1001 \\ \hline 1100 \quad 1100 \\ \hline 0110 \quad 0110 \\ \hline 10101110010 \\ \hline 001100+1 \\ \hline 0101 \quad 0011 \\ \hline 5 \quad 3 \end{array}$$

89-54 using 9's complement

$$\begin{array}{r} 89 \rightarrow 1000 \quad 1001 \\ -54 \rightarrow 0100 \quad 0101 \\ \hline 1100 \quad 1110 \\ \hline 0110 \quad 0110 \\ \hline 10011 \quad 0100 \\ \hline +1 \\ \hline 0011 \quad 0101 \\ \hline 3 \quad 5 \end{array}$$

X5-3 Arithmetic

X5-3 addition

1. Represent the given numbers in x5-3 form

2. Add x5-3 number by adding four bit group in each column

$$\begin{array}{r} 1000 \\ 1000 \\ \hline 2000 \end{array}$$

$$\begin{array}{r} \text{arithmetic} \\ \text{one} \\ \hline \text{two} \end{array} \quad \begin{array}{r} \text{one} \\ \text{one} \\ \hline \text{two} \end{array} \quad \begin{array}{r} \text{one} \\ \text{one} \\ \hline \text{two} \end{array} \quad \begin{array}{r} \text{one} \\ \text{one} \\ \hline \text{two} \end{array}$$

$6 \rightarrow 1001001100111$
 $3 \rightarrow 0110011010$
 $(-) \quad 0110 \quad 0011$
 10101
 1001
 $(-) \quad 11010 \rightarrow 9$

 $12 + 20$

$12 \rightarrow 0001 \quad 0010 \quad 0100 \quad 0101$
 $20 \rightarrow 0010 \quad 0000 \quad 0101 \quad 0011$
 $\hline 0011 \quad 0010 \quad 1001 \quad 1000$
 $\hline 0011 \quad 0011 \quad 0011 \quad 0011$
 $\hline 1 \quad 0110 \quad 0101$
 $\downarrow \quad \downarrow$
 $3 \ln x - 5 \quad 2 \ln x - 5$

 $37 + 28$

$37 \rightarrow 0110 \quad 01010$
 $28 \rightarrow 0101 \quad 1011$

 $1100 \quad 0101$
 0811

X-5-3 subtraction using complements method

Subtract $24_{10} - 10_{10}$ using 9's complement form

$$\begin{array}{r} 24 \xrightarrow{\times 5-3} \begin{array}{cc} 0101 & 0111 \end{array} \\ 99 \xrightarrow{\times 5-3} \begin{array}{cc} 1011 & 1100 \end{array} \\ \hline 1000110011 \\ 00110011 \\ \hline 101000110 \\ \hline 01000111 \\ \hline 1 \quad 4 \end{array}$$

9's complement of 10 is 99
 $\begin{array}{r} 99 \\ -10 \\ \hline 89 \end{array}$

$$\begin{array}{r} 24 \xrightarrow{\times 5-3} \begin{array}{cc} 0101 & 0111 \end{array} \\ 90 \rightarrow \begin{array}{cc} 1100 & 0000 \end{array} \\ \hline 1000110010 \\ (+)0011 (+)0011 \\ \hline 101000111 \\ \text{discards} \quad 1 \quad 4 \end{array}$$

37 - 28 using 10's complement form

$$\begin{array}{r} 37 \xrightarrow{\times 5-3} \begin{array}{cc} 0110 & 1010 \end{array} \\ 72 \xrightarrow{\times 5-3} \begin{array}{cc} 1010 & 0101 \end{array} \\ \hline 100001111 \\ (+)0011 (+)0011 \\ \hline 100111100 \\ \hline 0 \quad 9 \end{array}$$

10's complement of 28 is 72

246 - 592 using 10's complement form

$$\begin{array}{r} 246 \rightarrow \begin{array}{ccc} 0101 & 0111 & 1001 \end{array} \\ 408 \rightarrow \begin{array}{ccc} 0111 & 0011 & 1011 \end{array} \\ \hline 1100101110100 \\ 001100110011 (+) \\ \hline 1001100001111 \end{array}$$

9's complement of 1001 1000 0111 is 0110 0111 1001
 $\begin{array}{r} 3 \quad 4 \quad 6 \end{array}$

687-348 using 9's complement

$$\begin{array}{r}
 687 \rightarrow \begin{array}{ccc} 1001 & 1011 & 1010 \\ \hline 1001 & 1000 & 0600 \\ 1001 & 1001 & 1110 \\ \hline (+)0011 & (+)0011 & (-)0011 \\ \hline 10110 & 0110 & 1011 \\ \hline 3 & 3 & 9+1 \\ \hline 0110 & 0110 & 1100 \\ \hline 3 & 3 & 9 \end{array} \\
 \end{array}$$

354-672 using 10's complement

$$\begin{array}{r}
 354 \rightarrow \begin{array}{ccc} 0110 & 1000 & 0111 \\ \hline 0110 & 0101 & 1011 \\ \hline 1100 & 1110 & 10010 \\ \hline 0011 & 0011 & 0011 \\ \hline (-) & (-) & (+) \\ \hline 1001 & 1011 & 0101 \end{array} \\
 \end{array}$$

2's complement of 1001 1011 0101 is 0110 0100 1011

Grey code:

- non-weighted code
- unit distance code
- Reflexive code
- Used in data acquisition, A/D converters etc
- Not used for arithmetic operations

Binary number $\rightarrow B_n B_{n-1} B_{n-2} \dots B_1 B_0$

Grey number $\rightarrow G_n G_{n-1} G_{n-2} \dots G_1 G_0$

Binary to grey code conversion

$$G_n = B_n$$

$$G_{n-1} = B_n \oplus B_{n-1}$$

$$G_{n-2} = B_{n-1} \oplus B_{n-2}$$

$$G_1 = B_2 \oplus B_1$$

Grey to Binary conversion

$$B_n = G_n$$

$$B_{n-1} = B_n \oplus G_{n-1}$$

$$B_{n-2} = B_{n-1} \oplus G_{n-2}$$

...

$$B_1 = B_2 \oplus G_1$$

B → G

G → B

Binary → 1010

Grey → 1111

Grey → 1111

Binary → 1010

convert into gray code

i) 25₁₀ to 56₁₀

Draw the tabular form of conversion of 3 bit binary to

gray code

and also 4 bit binary to gray code

Binary $B_3 B_2 B_1$	Gray $G_3 G_2 G_1$
000	000
001	001
010	011
011	010
100	110
101	111
110	101
111	100

Binary $B_4 B_3 B_2 B_1$	Gray $G_4 G_3 G_2 G_1$
0000	0000 → 0
0001	0001 → 1
0010	0011 → 3
0011	0010 → 2
0100	0110 → 6
0101	0111 → 7
0110	0101 → 5
0111	0100 → 4
1000	1100 → 12
1001	1101 → 13
1010	1011 → 15
1011	1110 → 14
1100	1010 → 10
1101	1011 → 11
1110	1001 → 9
1111	1000 → 8